Rigorous 3D error analysis of kinematic scanning LIDAR systems

Craig Glennie, Ph.D., P. Eng.

Abstract. To date, LIDAR sensors have been primarily airborne, as utilized as a fast and efficient means of collecting topographic information. As a result, in research studies and in most commercial work the accuracy of the LIDAR information is primarily obtained by examining the vertical component of LIDAR error only. However, more and more end users are using LIDAR intensity maps to produce planimetric feature maps, and there are also emerging ground based kinematic laser scanning systems which are mounted on a van or truck type platform. For both of these uses, the traditional vertical only error analysis of the LIDAR system is inadequate when defining the overall expected accuracy of the end-product received from the system. Therefore, in order to quantify the overall 3D expected accuracy of LIDAR systems (both land and air based) a rigorous 1st order error analysis of the LIDAR georeferencing equations are undertaken. Typical error parameters are then placed into the error analysis to generate expected horizontal and vertical system accuracies for different LIDAR system configurations. Finally, the results obtained from the theoretical error analysis are independently verified using real world LIDAR data.

Introduction

Laser scanning formulas

Calculation of ground coordinates for objects from laser scanning system observations have been well documented in the literature, see Baltsavias (1999) for example. Coordinates on the ground can be calculated by combining the information from the laser scanner, integrated GPS/INS navigation system and calibrated values. The target coordinate equation is given as:

$$p_G^l = p_{GPS}^l + R_b^l \cdot R_s^b \cdot r^s - R_b^l \cdot l^b \qquad (1.1)$$

where:

- p_G^l coordinates of target point in local level (l) frame,
- p_{GPS}^l coordinates of navigation sensor center in l frame,
- R_b^l rotation matrix from body (b) frame or navigation frame to local level frame, defined by the three rotation angles roll, pitch and yaw,
- R_s^b rotation from laser scanner (s) frame into body frame, usually referred to as boresight matrix,
- r^s coordinates of target point given in laser scanner frame,
- l^b lever-arm from scanner origin to navigation center origin given in the body frame.

In examining equation (1.1), it becomes evident that all terms on the right hand side of the equation contain errors. Therefore, we can alternatively express the equation as:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_G^l = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GPS}^l + R_b^l(\omega \quad \varphi \quad \kappa) \cdot \left(R_s^b(d\omega \quad d\varphi \quad d\kappa) \cdot r^s(\alpha \quad d) - \begin{bmatrix} l_x \\ l_y \\ l_z \end{bmatrix}^b \right). \qquad (1.2)$$

The above formula shows that the ground coordinate calculated for the laser return is dependent upon 14 observed parameters. The 14 parameters are:

- X(1), Y(2), Z(3) location of the navigation sensor. These values are given by the DGPS navigation subsystem.
- ω(4), φ(5), κ(6) are the roll, pitch and yaw of the sensor w.r.t. the local level frame. These values are given by the IMU navigation subsystem.
- dω(7), dφ(8), dκ(9), are the boresight angles which align the scanner frame with the IMU body frame. These values must be determined by a system boresight calibration, see, e.g. Morin (2002) or Toth (2002).
- α(10) and d(11) are the scan angle and range measured and returned by the laser scanner assembly
- l_x(12), l_y(13), l_z(14) are the lever arm offsets from the navigation origin (IMU origin) to the measurement origin of the laser scan assembly. These values must be determined by measurement or system calibration.

Equation (1.2) is obviously non-linear. The most common method of examining the effects of errors in parameters is to linearize the formula by truncating a Taylor series expansion after the first term. As a result, the effect of small differential errors in the measured parameters can be observed on the output ground coordinates by the solution of a set of linear equations. Differentiating equation (1.2) w.r.t. the fourteen unknowns above leads to the general error formula:

$$\begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix}_G^l = \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix}_{GPS}^l + J \begin{bmatrix} \delta\omega \\ \delta\varphi \\ \delta\kappa \end{bmatrix} + K \begin{bmatrix} \delta\, d\omega \\ \delta\, d\varphi \\ \delta\, d\kappa \end{bmatrix} + B \begin{bmatrix} \delta\alpha \\ \delta d \end{bmatrix} + C \begin{bmatrix} \delta l_x \\ \delta l_y \\ \delta l_z \end{bmatrix}. \qquad (1.3)$$

The matrices, J, K, B, and C are the so-called Jacobians of the transformation, and are defined as:

$$J = \left[\frac{\delta p_G^l}{\delta \omega} \quad \frac{\delta p_G^l}{\delta \varphi} \quad \frac{\delta p_G^l}{\delta \kappa} \right], \quad K = \left[\frac{\delta p_G^l}{\delta\, d\omega} \quad \frac{\delta p_G^l}{\delta\, d\varphi} \quad \frac{\delta p_G^l}{\delta\, d\kappa} \right],$$

$$B = \left[\frac{\delta p_G^l}{\delta \alpha} \quad \frac{\delta p_G^l}{\delta d} \right], \quad C = \left[\frac{\delta p_G^l}{\delta l_x} \quad \frac{\delta p_G^l}{\delta l_y} \quad \frac{\delta p_G^l}{\delta l_z} \right]. \qquad (1.4)$$

Typical size of error parameters

Now that we have defined a methodology for examining the effect the observed parameters have on the determined ground coordinates, we must determine the level of expected errors in each of the observations. Therefore, we will examine and discuss typical error sizes for each group of observations. Unless otherwise specified, all error values quoted are assumed to be one sigma values.

IMU attitude errors

The inertial navigation component of the LIDAR system delivers the roll, pitch and yaw angles which rotate the measurements from the body frame of the vehicle into the mapping frame. Typically, the IMU components for LIDAR systems are bought as commercial off the shelf (COTS) systems from 2 or 3 different system manufacturers. As a result, it is fairly easy to determine typical accuracy specifications for the IMU subsystems by examining the manufacturer's technical specifications. It should also be noted that errors in LIDAR return position due to attitude errors are directly proportional to the range from scanner to target. As a result, a higher accuracy IMU is normally required for fixed wing operations compared to helicopter or ground based data collection due to the increased target range. The table below lists typical post-processed IMU attitude accuracies for various systems. Note that in all cases these accuracies assume sufficiently accurate DGPS coverage to be able to reliably estimate the biases and drifts of the inertial sensors.

Table 1: Typical IMU attitude accuracy specifications (Sources: http://www.novatel.com, http://www.applanix.com)

	Roll & Pitch (deg)	Heading (deg)
Short Range LIDAR		
Applanix 310	0.015	0.035
Novatel Spans (HG1700 AG58)	0.015	0.05
Long Range LIDAR		
Applanix 510	0.005	0.008
Applanix 610	0.0025	0.005

Boresight errors

There are a variety of approaches to boresight angle determination, but in general all of the approaches reduce to two fundamental ways of solution for the unkown angles. Both approaches take advantage of overlapping LIDAR strips, usually flown in different direction and sometimes with different flight elevations. The two approaches are:

1. Manual adjustment. The three boresight angles are manually adjusted, and the data is reprocessed until opposing passes visually line up. Normally, the edges of building(s) are used to visually line up the data. Process can be fairly time intensive and is highly dependent upon the skill and visual bias of the operator performing the adjustment.
2. Least Squares Adjustment. Tie point and/or control point observations between overlapping LIDAR strips are collected, and then run through a least squares adjustment to determine the best fit boresight angles. This approach is detailed in Toth (2002), Morin (2002) and Talaya et al. (2004).

In the author's experience at Terrapoint and elsewhere, the accuracy of the manual adjustment for boresight angles normally is no better than the accuracy of the IMU used to measure attitude. Using the least squares approach (which Terrapoint has implemented), statistics on boresight angle accuracy can be determined from the least squares adjustment. Accuracies on the level of 0.001° in roll and pitch, and 0.004° in yaw are routinely observed. This level of accuracy seems to agree fairly well with that shown in Morin (2002). Therefore, for the purposes of the error analysis, two sets of boresight errors will be simulated. The values which will be used are given in Table 2.

Table 2: Typical boresight angle determination errors

	Roll & Pitch (deg)	Heading (deg)
Manual Boresight	0.005	0.008
Least Squares Boresight	0.001	0.004

Laser scanner errors

There are a number of factors which effect the accuracy with which the laser scanner subassembly is able to measure the angle and distance from the LIDAR system to the ground target. A detailed discussion of these error sources can be found in the literature, see Morin (2002) for example. For the purposes of our error analysis we will reduce the error sources to errors in distance and errors in angles. We make this reduction because most laser scanner manufacturers quote their expected accuracy in terms of these two macro error components and do not specify the individual factors which contribute to the overall error. If need be, for a more rigorous analysis, the error model in equation (1.3) could be expanded to include additional terms for the laser scanner. The error in

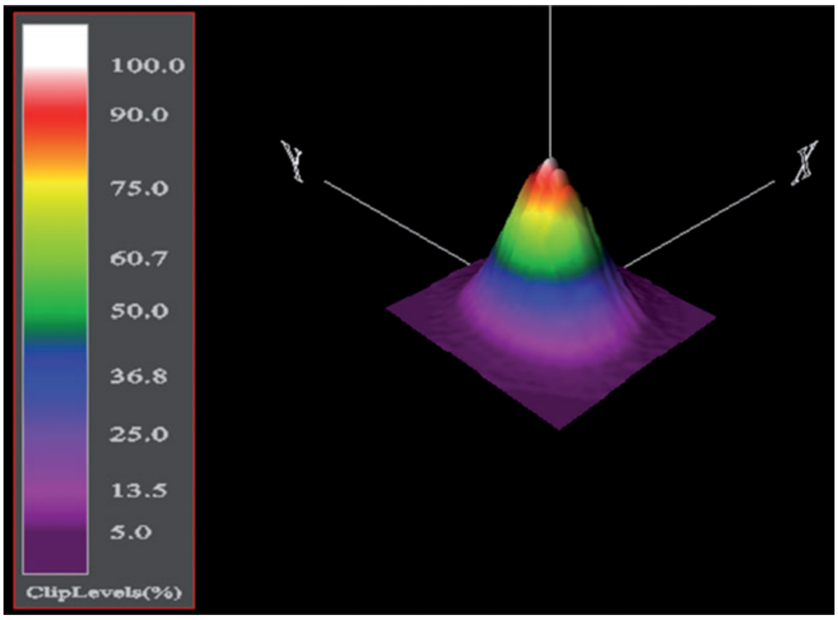

Figure 1: Relative Power Distribution of Output Pulse From Terrapoint ALTM LIDAR System

distance is normally just a function of the internal accuracy of the clock utilized to measure the time of flight of the laser pulse. Most air borne laser scanners typically quote single shot accuracies at the 1 to 2 cm level. We will use the more pessimistic 2 cm value. For our analysis here, angular measurement errors are considered to be a result of two error sources: (1), the angular resolution of the laser scanner angle encoder, and (2) uncertainty due to beam divergence. The first error source is straightforward, however, the second probably requires more discussion. The divergence of the laser beam gives rise to uncertainty in location of the actual point of range measurement. The instrument will record the apparent position of the point as along the emitted beam centerline, however the actual location is uncertain and could be anywhere within the beam footprint. A good demonstration of this uncertainty is given in Lichti and Gordon (2004), where they also demonstrate the anticipated level of uncertainty due to beam divergence can be quantified as (at a 1σ level) equal to one-quarter of the laser beam diameter in angular units. This estimate of uncertainty assumes a uniform level of laser power across the entire beam width diameter, which is not typically the case. Figure 1 shows a typical power distribution of an outgoing laser pulse from Terrapoint's ALTM high range LIDAR system. As the Figure shows, the power across the pulse is not uniform, and has a very definite peak and slope. This would suggest that the level of uncertainty is probably a little bit less than $\frac{1}{4}$ of the beam divergence since with the greater power near the center of the beam would increase the probability of a return from nearer to the center of the emitted beam centerline. However, to purposely err on the side of caution, we will use the $\frac{1}{4}$ value. This seems to be a reasonable assumption since we will be neglecting the effect of incidence angle, and terrain slope, which have been shown to be significant sources of horizontal and vertical error, see Morin(2002) for example.

The table below lists typical ranging and angular measurement accuracies for various altitude classes of LIDAR sensors.

Lever-arm offset errors

It is quite evident that the center of observations from the laser scanner, and the origin of the navigation subsystems cannot be co-located. Therefore, the precise offset or lever-arm between the two centers must be known in order to accurately georeference the laser scanner measurements. Since the physical measurement origin of the navigation system or laser scanner assembly cannot be directly observed, the lever-arm offset must be obtained indirectly. There are two common methods of obtaining these offsets. The first employs a calibration procedure (i.e. making measurements of known points) to determine, among other parameters, the lever-arm offset of the laser scanner. In practice however, the lever-arm components are fairly weakly observable due to their high correlation with other error sources within the system (specifically boresight values, DGPS errors, and IMU to GPS lever arm errors). The second method of offset determination is by a combination of physical measurement (using a tape measure) and use of the engineering drawing supplied for the IMU and laser scanner. Obviously, the second method is much simpler to implement, and is therefore used in a majority of cases. This approach too has its sources of error, because it assumes that the IMU and laser scan axes are aligned, and that the drawings accurately represent the origin of the subassemblies. Therefore, as a conservative estimate herein, and

(AutoPDF V7 17/8/07 11:01) WDG (210×297mm) TMath J-1817 JAG, 1:3 PMU: A(A) 2/8/2007 PMU: WSL(W) 16/08/2007 pp. 147–157 1817_1-3_17-C (p. 149)

based on the author's previous experience, we will assume that the lever arm offset can be measured with an accuracy of 2 centimeters in all three components. It is also assumed that the IMU and laser have been rigidly mounted to a common frame so that no differential motion between their measurement origins can occur during data capture.

Positioning errors

The absolute expected level of DGPS kinematic positioning errors for a LIDAR survey is normally fairly difficult to quantify. In general, there are a number of factors that have a direct impact on the resultant positioning accuracy of the DGPS subsystem. These factors, such as atmospheric errors, multipath, poor satellite geometry, baseline length, and loss of lock, are difficult to predict and therefore do not lend themselves to a generic error model. A good rule of thumb for relative DGPS kinematic positioning that the author has often used is that the positioning accuracy for relatively short (<30 km) kinematic baselines is on the order of 2 cm + 1 PPM horizontally, and 2 cm + 1 PPM vertically. This accuracy level assumes no loss of lock of GPS signals, good satellite geometry, minimal multipath, and low ionospheric activity. Obviously, applying a generic accuracy level to the ground based system is even harder due to the frequent expected masking of GPS signals, by buildings, vegetation and other line of sight obstructions. For an excellent discussion on DGPS error sources, the reader is referred to Raquet (1998). In addition, Bruton (2000) also provides a detailed examination of DGPS error sources for precise airborne positioning.

In examining equation (1.3) it is noted that the navigation system positioning errors have a direct impact on the resultant position of the LIDAR point cloud which is not dependent upon any of the other observational parameters. Because this relationship is direct, and because there is a great deal of uncertainty in the magnitude of the GPS positioning errors, they are not included in the following analysis. For their own reference, the reader is encouraged to use a DGPS error budget number that they consider adequate for their survey specifications, and simply add that amount to the results presented below.

Theoretical accuracy analysis

A first-order error analysis using equation (1.3) requires some assumptions to be made about system dynamics and typical magnitudes of various fixed parameters. Specifically, the items at issue are:

- Typical dynamics (i.e. magnitude of roll pitch and yaw) for the platform during data acquisition,
- Expected values for range and angle measurements,
- Magnitude of actual boresight angles,
- Normal laser scanner to IMU lever arm offsets.

Therefore, in an attempt to make the analysis as realistic as possible actual collection dynamics, for a fixed wing, helicopter and ground based platform were used, along with actual range and angle measurements for all three platforms. For the fixed wing platform, the scan angle field of view (FOV) was $\pm 18^\circ$, and for the helicopter and ground based platforms the FOV was $\pm 30^\circ$. Normal boresight angles for an airborne platform are usually close to 0°, so a pessimistic value of 2° in all three axis was used for the analysis. For the ground-based platform, the boresight angles can take on nearly any value, and as a result, here, the worse case scenario was assumed and all boresight angles were set at 45°. Finally, the lever arm between the scanner and IMU was set at 0.50 m in all three axis. This again, is probably larger than normally observed, but would err on the pessimistic side.

Fixed wing LIDAR

For most of the commercially available high range LIDAR sensors, the laser scanner errors (i.e. angular error and ranging error) are fairly similar, and are not something easily improved by the end user. Therefore, for the basis of comparison, and because it is the most common commercially used system, the specifications for the Optech 3100 from Table 3 will

Table 3: Representative laser scanner range and angle accuracy specifications (sources: www.optech.ca, www.riegl.com, www.leica-geosystems.com)

SENSOR TYPE	Range Error (m)	Angular Resolution (°)	Beam Div. (1/e) (mRad)	Beam Angular Uncertainty (°)	Total Angular Error (°)
Long Range					
Optech 3100	0.02	0.001	0.3	0.0043	0.0044
Leica ALS50-II	0.02	0.001	0.15	0.00215	0.0024
Riegl Q-560	0.02	0.001	0.5	0.00716	0.0073
Mid. Range					
Riegl Q-240	0.02	0.005	2.7	0.0387	0.039
Riegl Q-280	0.02	0.0025	0.5	0.00716	0.0076
Short Range	0.02	0.01	2.7	0.0387	0.04

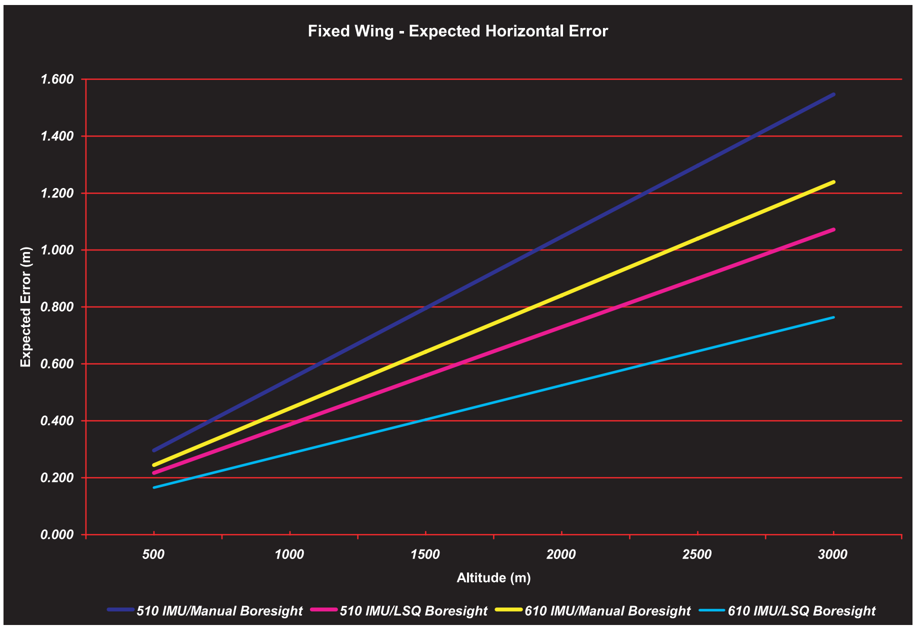

Figure 2: Fixed Wing Horizontal Errors, Various Scenarios

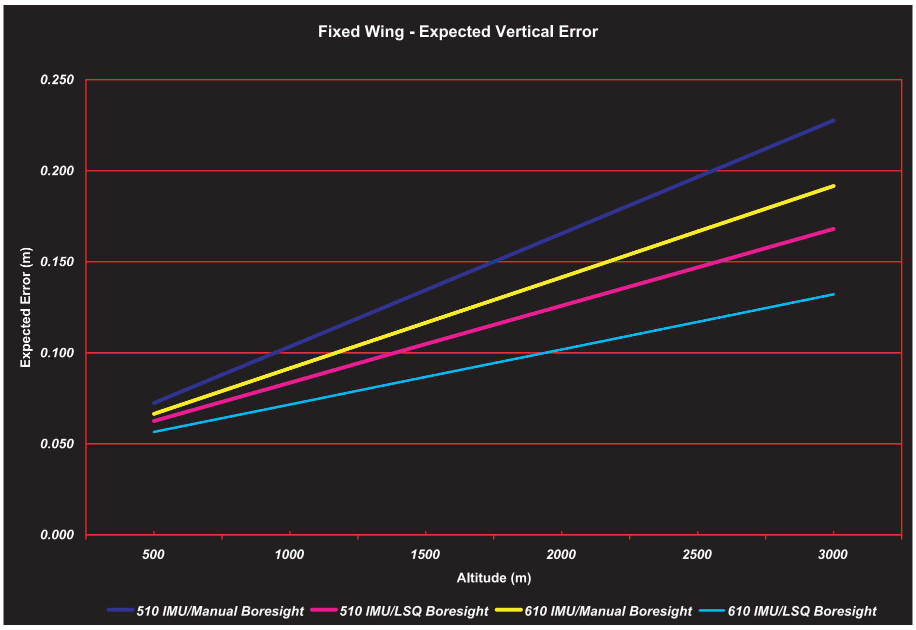

Figure 3: Fixed Wing Vertical Errors, Various Scenarios

be used. However, the end user does have a choice when it comes to the inertial system integrated with the sensor, and in the methodology of boresight calibration for the system. At the present time, most commercial high range LIDAR sensors are integrated with an IMU with performance characteristics similar to the Applanix 510 IMU (Table 1). However, a couple of manufacturers (Applanix and IMAR Navigation) have begun to offer IMUs with better accuracy specifications (see Applanix 610 in Table 1). Therefore, for comparison purposes, four simulations have been run for the fixed wing system:

1. Manual Boresight, 510 Accuracy Class IMU,
2. Least Squares Boresight, 510 Accuracy Class IMU,
3. Manual Boresight, 610 Accuracy Class IMU,
4. Least Squares Boresight, 610 Accuracy Class IMU.

The results of the four runs are shown in Figures 2 and 3. Again note, per the discussion above, GPS error sources have not been included in these (or subsequent) graphs.

By examining Figure 2, it is quite clear that the

(AutoPDF V7 17/8/07 11:01) WDG (210×297mm) TMath J-1817 JAG, 1:3 PMU: A(A) 2/8/2007 PMU: WSL(W) 16/08/2007 pp. 147–157 1817_1-3_17-C (p. 151)

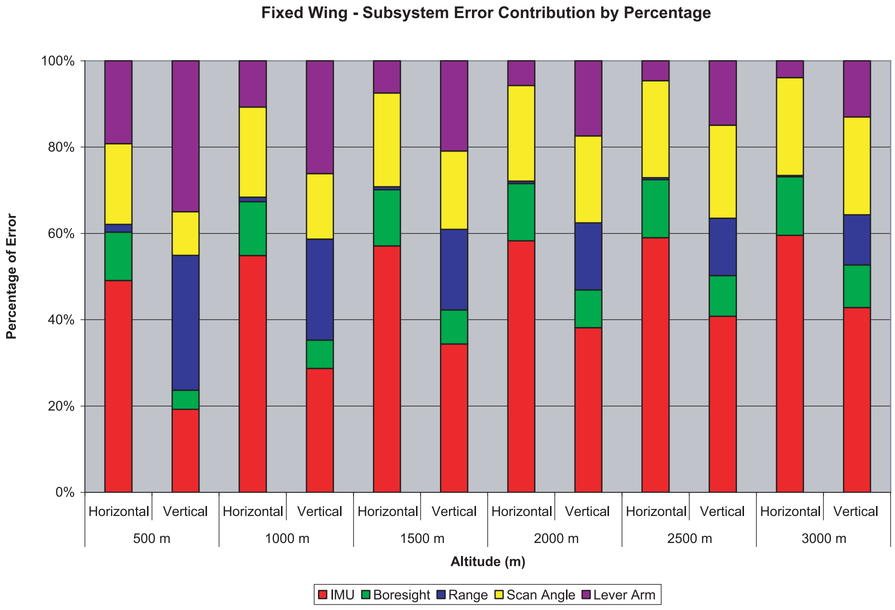

Figure 4: Fixed Wing System, Subsystem Error Contribution by Percentage (Applanix 510 IMU, Least Squares Boresight)

method of boresighting and the class of IMU used has a significant impact on the horizontal system accuracy. The impact increases the higher the system flies, which is expected since angular errors increase proportionally to distance. An examination of Figure 3 shows that the increased accuracy of IMU and boresighting is not as pronounced on overall vertical error. At low flight heights (<1000 m), the increased vertical accuracy is only on the order of at best 5 cm. Currently in industry, evaluation of fixed wing LIDAR data has mostly focused on vertical system accuracy, and has largely ignored the horizontal component. However recently, with the use of intensity maps for planimetric feature mapping, and with the emergence of Lidargrammetry (Geocue, Patent Pending) for LIDAR data editing and feature extraction, there should be a new focus on horizontal LIDAR accuracy. Clearly, LIDAR users will have to pay increased attention to the accuracy class of the IMU, and to the method of boresighting used for the system if they are to feel confident in the horizontal accuracy achieved. To this end, perhaps a boresight calibration report with associated confidence statistics should be considered a standard deliverable with any planimetric features identified from high-range LIDAR.

Now that we have looked at the overall, expected system accuracy, it is also useful to look at the error contributions of different categories. Figure 4 contains error contributions by percentage for the IMU, boresight angles, range, scan angle, and lever arm errors.

The horizontal errors for fixed wing LIDAR are clearly dominated by attitude errors, the combined IMU error and boresighting error contribute from 60% to 75% of the overall horizontal error depending upon flight elevation. The vertical channel also shows a significant amount of error due to these two factors. Here, the attitude errors contribute 25% to over 50% of the error dependent upon altitude.

The results presented in Figure 4 clearly demonstrate two points: (1) a least squares boresighting with sufficient redundancy and control to achieve millidegree accuracy is a must, and (2) further high range LIDAR accuracy improvements will be highly dependent upon improvements in attitude determination from IMU advancements, or from integration of other attitude sensors with IMUs.

To date, most accuracy evaluations and contract specifications for high range LIDAR focus only on vertical accuracy. The question arises therefore, for a given vertical accuracy, what is the corresponding horizontal system accuracy. In comparing Figures 2 and 3 it is obvious that the horizontal errors are significantly larger than the vertical. Therefore, to examine the relationship between horizontal and vertical errors the ratio between the two was computed for the four different systems configurations examined. The results are presented in Table 4. By examining the results presented, a good general rule of thumb seems to be that the horizontal accuracy is at least 5 times worse than the expected vertical accuracy.

Helicopter based LIDAR

The above section on fixed-wing LIDAR clearly showed the importance of a good boresight results for overall system performance. Therefore, for the helicopter platform analysis, we will concentrate on examining the effects of different IMU classes and laser scanners, and will assume we have very good

(AutoPDF V7 17/8/07 11:01) WDG (210×297mm) TMath J-1817 JAG, 1:3 PMU: A(A) 2/8/2007 PMU: WSL(W) 16/08/2007 pp. 147–157 1817_1-3_17-C (p. 152)

Table 4: Fixed wing: ratio of horizontal to vertical errors by flight altitude

Flight Height	510/Manual Boresight	510/LSQ Boresight	610/Manual Boresight	610/LSQ Boresight
500	4.08	3.46	3.68	2.92
1000	5.27	4.63	4.84	3.97
1500	5.92	5.33	5.51	4.66
2000	6.32	5.79	5.94	5.14
2500	6.60	6.13	6.24	5.50
3000	6.80	6.37	6.47	5.78

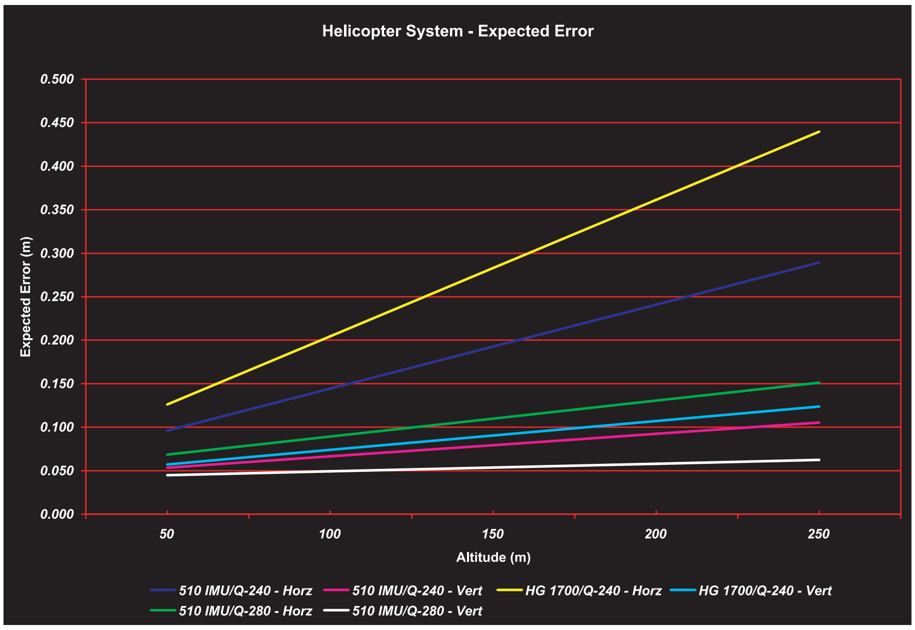

Figure 5: Helicoptor System Horizontal and Vertical Errors for Three Different Scenarios

boresight results. Three different error analyses will be run:

1. HG1700 IMU, with Q-240 Laser, baseline analysis with common helicopter LIDAR system components
2. Applanix 510 IMU, with Q-240 Laser, to show the effect of better IMU performance
3. Applanix 510 IMU, with Q-280 Laser, to highlight the advantages of a smaller beam divergence.

Figure 5 shows helicopter system expected accuracy for the above three test cases.

The results in Figure 5 clearly show that the significant increase in accuracy class of the IMU has almost no effect on the overall vertical error budget (1.8 cm max @ 250 m AGL). However, the better IMU does improve the the horizontal error by significant amount. The reduced laser beam divergence (Riegl Q-280) has a more significant effect on the vertical accuracy (4.3 cm at 250 m AGL), and shows a almost factor of 2 improvement in horizontal accuracy. Therefore, it is quite evident that the beam divergence is a very significant factor to overall system accuracy. To further emphasize this, a breakdown of the error contribution by category for a helicopter system with a 510 class IMU and Q-240 laser scanner is given in Figure 5.

Figure 6, shows that the error breakdown of a helicopter system is significantly different from that of a fixed wing system (compare to Figure 4). The dominant error in Figure 6 is clearly the scanner angle error, which is largely a result of the large beam divergence for that laser. As a comparison, Figure 7 displays the same error breakdown, but using the Q-280 laser, which has significantly less beam divergence (0.5 mRad vs. 2.7 mRad).

The error contributions by category in Figure 7 are more evenly distributed and are not nearly as highly weighted on the attitude errors. This is directly a result of the in general lower flight altitude of these systems. For the Q-280 helicopter system, the largest error contribution seems to be coming from the lever-arm errors, which were assumed to be 2 cm in all three axes. This is encouraging, because the 2 cm figure was relatively pessimistic, and it is possible to significantly reduce lever-arm error by careful measurements or perhaps a special calibration procedure.

Finally, for a helicopter-based system, an examination of the ratio between horizontal and vertical errors was undertaken. The results are presented in

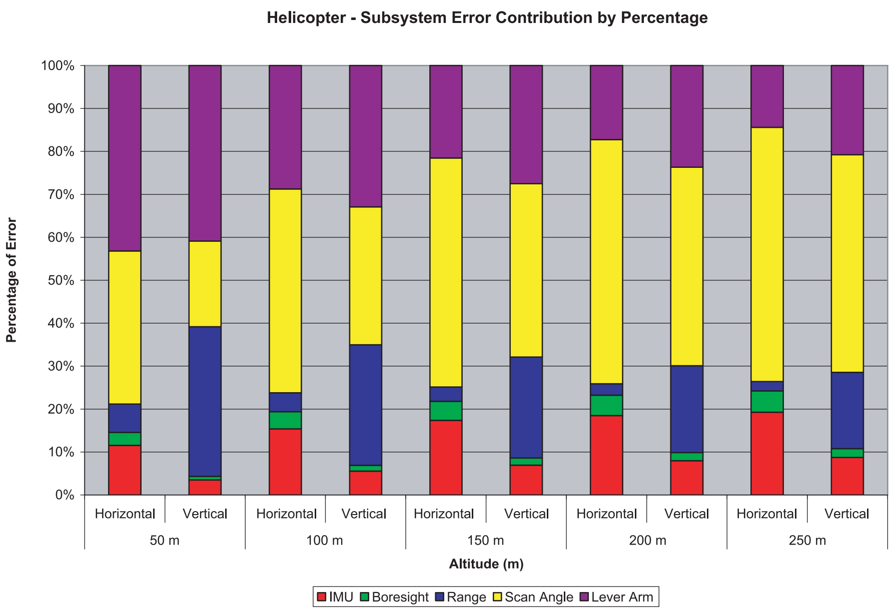

Figure 6: Helicopter, Subsystem Error Contribution by Percentage (Applanix 510 IMU, Q-240 Laser)

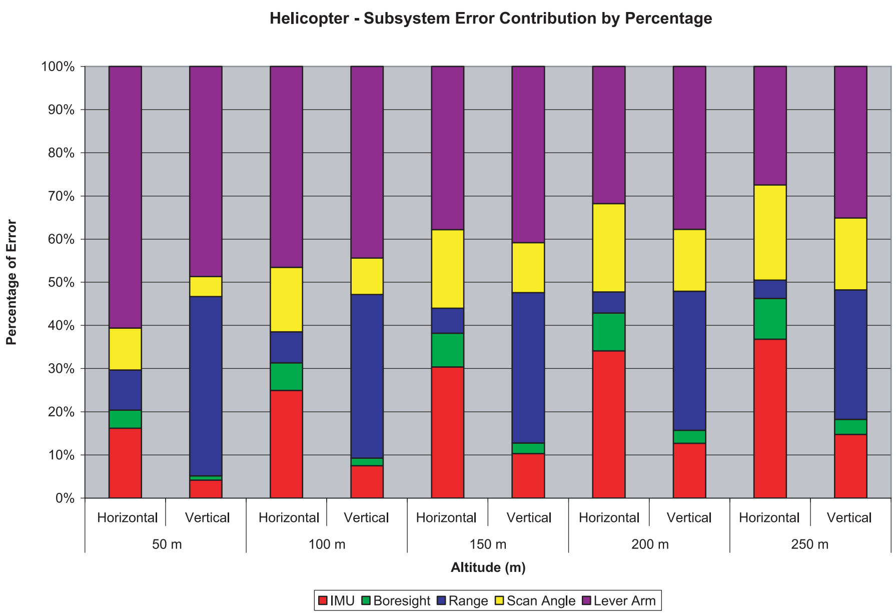

Figure 7: Helicopter, Subsystem Error Contribution by Percentage (Applanix 510 IMU, Q-280 Laser)

Table 5. The ratio for a helicopter system of about 2 to 2.5 is dramatically better than that for a fixed wing platform. This would suggest that any high accuracy planimetric feature mapping performed from LIDAR data is probably best performed using data acquired from a helicopter platform.

Ground based kinematic LIDAR

For a final analysis, a ground based kinematic LIDAR system, similar to the one presented in Newby and Mrstik (2005) is analyzed. The analysis of the ground based system is unique mostly due to assumptions about the orientation of the LIDAR sensor(s) themselves. In general, in an airborne environment,

Table 5: Helicopter: ratio of horizontal to vertical errors by flight altitude

FLIGHT HEIGHT	510 IMU/ Q-240	HG1700 IMU/ Q-240	510 IMU/ Q-280
50	1.80	2.20	1.52
100	2.17	2.77	1.81
150	2.43	3.13	2.05
200	2.61	3.37	2.25
250	2.75	3.56	2.43

(AutoPDF V7 17/8/07 11:01) WDG (210×297mm) TMath J-1817 JAG, 1:3 PMU: A(A) 2/8/2007 PMU: WSL(W) 16/08/2007 pp. 147–157 1817_1-3_17-C (p. 154)

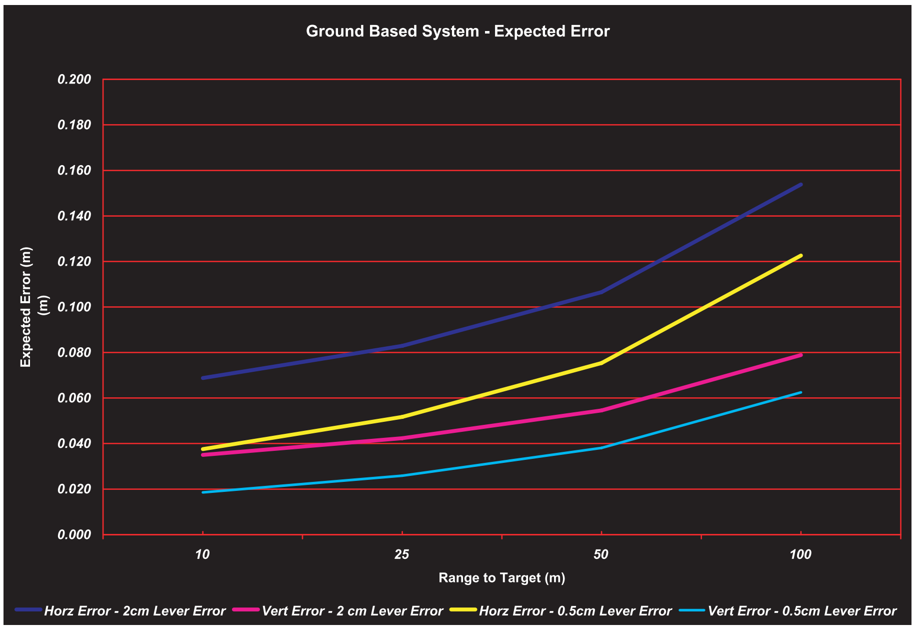

Figure 8: Ground Based System Horizontal and Vertical Errors for Two Different Lever-Arm Errors

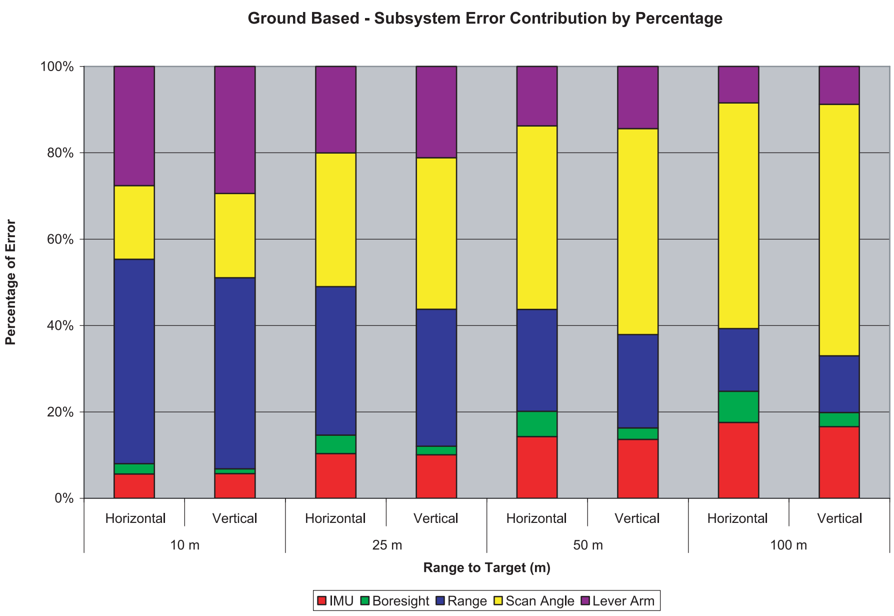

Figure 9: Ground System, Subsystem Error Contribution by Percentage (510 IMU, LSQ Boresight, 0.5 cm Lever Arm Error)

the laser scanner subassembly is normally oriented close to the axis of the IMU, in such a manner that the boresight misalignments are quite small (normally under a couple of degrees). However, for a ground based system, this assumption is invalid, as the scanner subassembly may be pointing in any number of orientations. Therefore, the results below are computed considering a worst case scenario boresight offset of 45° in all three axes, a good boresight solution is assumed, and a 510 IMU was simulated. To highlight the effect of varying lever arm errors, two different lever arm errors of 2 cm and 0.5 cm in all three axes are compared. The results are presented in Figure 8.

For the ground based system, the improvement in expected lever-arm errors behaves essentially like a constant improvement in both horizontal and vertical accuracy. This is to be expected, since, as formula 1.2 shows, the lever-arm error is not modulated by scan angle or range to target. A breakdown of the error contribution by category for a ground-based system with a 510 class IMU, and 0.5 cm lever-arm errors is given in Figure 9.

The error budget displayed in Figure 9 is dominated by laser scanner errors. The effect of attitude errors is significantly reduced (under 25% in all cases), due to the shorter range measurements normally taken from

(AutoPDF V7 17/8/07 11:01) WDG (210×297mm) TMath J-1817 JAG, 1:3 PMU: A(A) 2/8/2007 PMU: WSL(W) 16/08/2007 pp. 147–157 1817_1-3_17-C (p. 155)

a ground based platform. Therefore, it is obvious that further improvement to the ground scanning system is dependent upon accuracy improvements in the scanner assembly itself. The horizontal to vertical error ratio for the ground system, of approximately two, is also similar to that of the helicopter.

Comparison of analysis with real world results

Fixed wing system

In early 2006, one of Terrapoint's proprietary ALTMS was tested for both horizontal and vertical accuracy during production for an ongoing contract. Sixteen targets were established at an airport site, 8 reflective targets for horizontal analysis, and 8 vertical only targets. The calibration site was overflown with the LIDAR unit at the start and end of a LIDAR mission on eight separate missions. As a result, we have 16 independent flight lines, flown at 1000 meters AGL, for comparison with the targets. Vertical comparisons were obtained by comparing known elevations with that derived from a TIN of the LIDAR ground returns. Horizontal target locations were obtained by digitizing them using a 1 meter resolution raster intensity image created for each flight line.

To compare the ground truthing results, the specifications for the ALTMS system were also run through the error analysis performance. The ALTMS system has a 510 class IMU, a 0.75 mRad beam divergence, a 2 cm ranging error, and is boresighted using an optimization approach which has a known estimated accuracy. The GPS base station for all of the above test lines was less than 1 km from the airplane, and therefore 2 cm of error for both horizontal and vertical were added to the expected error values. The results of the error analysis, along with the comparison of the flight data with the ground control is summarized in Table 6.

Note that for the horizontal errors, two different values are displayed, a raw RMSE and a final RMSE. The horizontal target coordinates were digitized off of a raster intensity image with 1 meter pixels, and therefore some error due to the finite image resolution must be accounted for. As a result, the Final RMSE numbers take into account an error in digitization of $\pm\frac{1}{2}$ a pixel, and should be considered a more accurate estimation of overall horizontal system accuracy. Overall, the actual system results in comparison with ground targets show a very good level of agreement with the expected errors derived from the model, which validates the 1st order error analysis model.

Table 6: Horizontal and vertical comparison of ALTMS data with ground control, and expected errors based on 1st order error model

Value (Meters)	Easting	Northing	Horizontal	Vertical
Minimum	−1.807	−2.096	0.043	−0.202
Maximum	2.189	2.123	2.688	0.215
Average	−0.183	−0.094	0.775	−0.021
Raw RMSE	0.706	0.619	0.939	0.086
Final RMSE	0.499	0.365	**0.618**	**0.086**
Expected Errors (Model)			**0.553**	**0.105**

For further validation of the error model, it would be ideal to compare the expected errors from the model with actual results reported in other sources. At first thought this should be simple because accuracy studies of fixed wing LIDAR surveys are plentiful in the literature. However, a problem arises because the first order error model presented requires a good deal of initial *a priori* information in order to derive accuracy estimates (i.e. boresight methodology, flight height and dynamics, IMU specifications, laser scanner error characteristics). Unfortunately, in most sources, a majority of these parameters are not detailed, and therefore a direct comparison to the presented error model is not feasible without having to make too many assumptions.

Helicopter system

During a least-squares boresighting of a LIDAR system, statistics can be generated on the residual misclosure of measurements of control points and ties points. Assuming that all other significant systematic sources of error have been taken care of, the residual misclosure should give an estimate of the expected horizontal and vertical accuracy of the system for the specified flight conditions. Consequently, the results of a boresight adjustment can be used to independently verify the 1st order error analysis of a LIDAR system. Therefore, a boresight analysis of one of Terrapoint's helicopter based systems was undertaken. The system consists of a Riegl Q-140 laser with a Honeywell HG1700 IMU. A calibration site was overflown in four directions, at a height of 100 m AGL, and 115 tie points were collected from the data to determine the boresight angles of the system. The expected accuracy of the boresight angles from the least squares adjustment were also used as seed values for the 1st order accuracy analysis. Table 7 gives the final RMS misclosure errors of the tie point observations along with the expected errors based on the 1st order model.

The results in Table 7 show very good correlation between the expected errors and the misclosure ob-

Table 7: Helicopter boresight residual errors and expected errors based on 1st order model

Value (Meters)	Horizontal	Vertical
Final RMSE	0.292	0.089
Expected Error	0.250	0.095

Table 8: Ground system, vertical comparison with ground control (from Glennie et al. 2006)

Lidar V. Ground Truth	Value (Meters)
Average	−0.016
Maximum	0.101
Minimum	−0.105
Standard Deviation	0.058
RMS	0.059

served from the boresight adjustment. These results again seem to validate the performance of the 1st order error model.

Ground based system

In Glennie et al. (2006), the use of a ground based system with a Honeywell HG1700 IMU was detailed. Several tests of this system were performed in an area of dense ground control to provide an independent accuracy check of the system. In the test of the system described, typical ranges to target were between 10 to 25 meters. The GPS reference station for the test was less than 1 km away from the test area. The results of vertical comparison with ground control given in Glennie et al. (2006) are repeated in Table 8 below.

Running this system configuration through the error analysis software produces an expected vertical accuracy of 4 to 5 centimeters. Adding a GPS error of 2 cm in the vertical (using the 2 cm + 2 ppm rule-of-thumb given earlier) results in a expected vertical error of 6 to 7 centimeters, which agrees very well with the results in Table 8, and would appear to further validate the accuracy analysis results.

Conclusions and future work

A rigorous error model for kinematic laser scanning from airborne and ground based platforms was presented. Various scenarios for three different operating platforms were shown to highlight the effects of the major system error sources. Finally, real world data results were presented which validated the results obtained in the modeling process. Upgrades to hardware and processes to improve the accuracy results of the LIDAR systems can be simulated before purchases are made to verify and validate the expected accuracy improvements from the new hardware. In addition, operation can be planned to obtain data which meets specified horizontal and vertical accuracy requirements with a great degree of a priori confidence.

The results here, concentrated on accuracy of each individual LIDAR return, and did not attempt to model accuracy of the resultant surface model (e.g. DEM or DTM) obtained from the LIDAR data. This type of modeling is much more sophisticated, and requires additional consideration of factors such point density, thickness of vegetative cover, and terrain slope. Future research will attempt to model all of these factors to obtain an estimated accuracy map of the resultant LIDAR derived products.

References

Bruton, A. M., Improving the Accuracy and Resolution of SINS/DGPS Airborne Gravimetry, Ph.D. Thesis, The University of Calgary, UCGE Report #20145, 2000 (Available at: http://www.ensu.ucalgary.ca/links/GradTheses.html).

Baltsavias, E. P., Airborne laser scanning: basic relations and formulas, ISPRS Journal of Photogrammetry & Remote Sensing, 54 (1999), 199–214.

Glennie, C., K. Kusevic, and P. Mrstik, Performanc Analysis of a Kinematic Terrestrial LIDAR Scanning System, Proceedings of MAPPS/ASPRS Fall Conference, San Antonio, TX, November 6–10, 2006.

Lichti, D. D., and S. J. Gordon, Error Propagation in Directly Georeferenced Terrestrial Laser Scanner Point Clouds for Cultural Heritage Recording, Proceedings of FIG Working Week 2004, Athens, Greece, May 22–27, 2004.

Morin, K. W., Calibration of Airborne Laser Scanners, Masters Thesis, The University of Calgary, UCGE Report #20179, 2002 (Available at: http://www.ensu.ucalgary.ca/links/GradTheses.html).

Newby, S., and P. Mrstik, Lidar on the Level in Afghanistan. GPS World, 16:7 (2005),16–22.

Raquet, J. F., Development of a Method for Kinematic GPS-Carrier Phase Ambiguity Resolution Using Multiple Reference Receivers, Ph.D. Thesis, The University of Calgary, UCGE Report #20116, 1998 (Available at: http://www.ensu.ucalgary.ca/links/GradTheses.html).

Talaya, J., R. Alamus, E. Bosch, A. Serra, W. Kornus, A. Baron, Integration of a Terrestrial Laser Scanner with GPS/IMU Orientation Sensors, Proceedings of XXth ISPRS Congress, Commision 5, Istanbul, Turkey, July 12–23, 2004.

Toth, C. K., Calibrating Airborne LIDAR Systems, Proceedings of ISPRS Commission II Symposium, Xi'an, China, August 20–23, 2002, 475–480.

Received: Feb 1, 2007 Accepted: May 5, 2007

Author information
Craig Glennie, Ph.D., P. Eng.
Terrapoint USA Inc.
25216 Grogans Park Drive
The Woodlands, TX 77380
craig.glennie@terrapoint.com